# MHD TURBULENCE IN ACCRETION DISKS: THE IMPORTANCE OF THE MAGNETIC PRANDTL NUMBER

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Abstract. The magnetorotational instability (MRI) is the most likely source of MHD turbulence in accretion disks. Recently, it has been realized that microscopic diffusion coefficients (viscosity and resistivity) are important in determining the saturated state of the turbulence and thereby the rate of angular momentum transport. In this paper, we use a set of numerical simulations performed with a variety of numerical methods to investigate the dependance of α, the rate of angular momentum transport, on these coefficients. We show that α is an increasing function of the magnetic Prandtl number Pm, the ratio of viscosity over resistivity. In the absence of a mean field, we also find that MRI–induced MHD turbulence decays at low Pm.

1 Introduction

In accretion disks, turbulence is believed to be the main driver of outward angular momentum transport. For decades, its origin has been the central issue in accretion disk theory. It is now widely accepted that the MRI (Balbus & Hawley 1991) is the best candidate to play that role. During its nonlinear phase, its drives MHD turbulence and saturates at amplitudes compatible with the observations. However, recent numerical results (Fromang & Papaloizou 2007, Fromang et al. 2007, Lesur & Longaretti 2007) have cast doubt into the validity of earlier simulations. In particular, it has been shown that the saturated state of the turbulence depends on both the value of viscosity ν and resistivity η. In this paper, we study this dependance by using a set of local simulations performed with various numerical methods in the framework of the shearing box model.
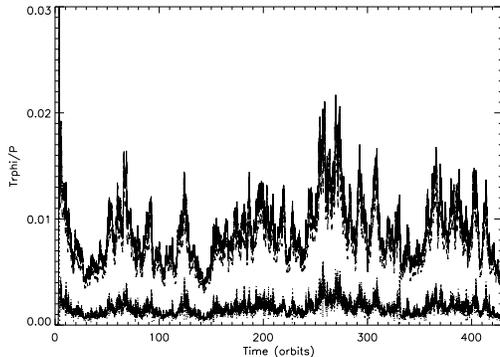
Fig. 1. Time history of the Reynolds stress (*lower dashed line*), Maxwell stress (*upper dashed line*) and total stress (*solid line*) in the case $Pm = 4$ and $Re = 3125$ as obtained with ZEUS. The time averaged value of the total stress is $\alpha = 9.1 \times 10^{-3}$.

2   A comparison case

Figure 1 shows the time history of the Reynolds stress (*lower dashed line*), Maxwell stress (*upper dashed line*) and total stress α (*solid line*) in a numerical simulations performed in the shearing box with the MHD code ZEUS (Hawley & Stone 1995). The mean field threading the box is zero. The resolution is $(Nx, Ny, Nz) = (128, 200, 128)$. The Reynolds number is $Re = c_s H / \nu = 3125$, where c_s is the sound speed (note that we use an isothermal equation of state) and H is the disk scaleheight. Finally, we set $Pm = 4$. Figure 1 shows that MHD turbulence is sustained in that case. The time averaged value of α is 9.1×10^{-3}.

The same simulation was repeated using three different codes, namely NIR-VANA (Ziegler & Yorke 1997), The Pencil code (Brandenburg & Dobler 2002) and a spectral code (Lesur & Longaretti 2007). The results of the four simulations are in very good agreement as the scatter in the time averaged α value is only of about 10% (Fromang et al. 2007). This showed that numerical dissipation does not affect the results shown in figure 1. The parameter space (Re, Pm) can thus be probed using such simulations.
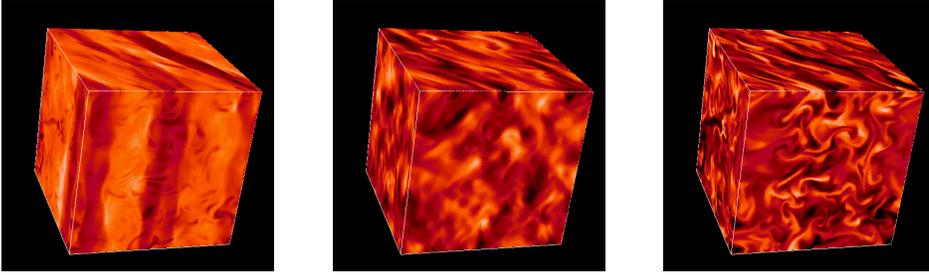
3   Parameter space study

In this section, using ZEUS, we vary the value of ν and η in order to study the dependance of α on the Reynolds and magnetic Prandtl numbers.

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Fig. 2. Illustration of the flow structure in the case $Pm = 4$ and $Re = 6250$. The left panel shows a snapshot of the density, the middle panel a snapshot of the vertical velocity and the right panel a snapshot of the toroidal component of the magnetic field.

3.1 The structure of the flow

To illustrate the flow structure, we start by showing in figure 2 snapshots of the density (*left panel*), vertical velocity (*middle panel*) and toroidal magnetic field (*right panel*). In this case, the resolution is $(256, 400, 256)$, $Re = 6250$ and $Pm = 4$. In agreement with earlier results (Papaloizou et al. 2004, Gardiner & Stone 2005), we find density wave propagating radially in the box. Note also that the size of the typical structures in the velocity tends to be larger than the size of the typical structures in the magnetic field. This is because $Pm \geq 1$: the viscous dissipation length is larger than the resistive dissipation length.

3.2 The effect of the Prandtl number

Figure 3 shows the time history of α in six simulations having $Pm = 16$ (*dotted–dashed line*), $Pm = 8$ (*dashed line*), $Pm = 4$ (*solid line*), $Pm = 2$ (*dotted line*) and $Pm = 1$ (*dotted–dotted–dashed line*). In all simulations, the resistivity is such that $Re_M = c_s H/\eta = 12500$. Figure 2 shows that α is an increasing function of Pm (it also highlights the importance of kinematic viscosity in determining the value of α as the value of η is kept fixed in these simulations). In addition, these results also demonstrate that MHD turbulence decays when $Pm \leq 2$.

3.3 The overall results

Figure 4 summarizes our results. It gives the nature of the flow in the (Re, Pm) plane: "YES" means that MHD turbulence is sustained while "NO" corresponds to cases in which turbulence decays. At each Re, there is a critical value of Pm, Pm_c, below which turbulence is not sustained. For the range of Re probed in these simulations, Pm_c is a decreasing function of Re. In addition, turbulence was never sustained when $Pm \leq 1$.
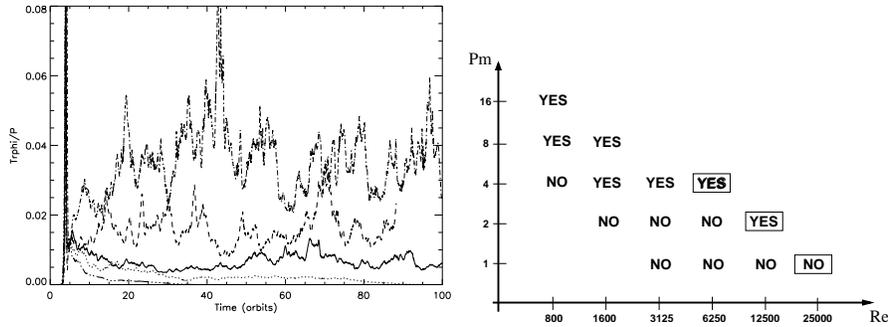
Fig. 3. Left panel: Time history of α in the case $Re_M = 12500$ and $Pm = 16$ (*dotted–dashed line*), $Pm = 8$ (*dashed line*), $Pm = 4$ (*solid line*), $Pm = 2$ (*dotted line*) and $Pm = 1$ (*dotted–dotted–dashed line*). The angular momentum transport increases with the magnetic Prandtl number Pm and vanishes when $Pm \leq 2$ for this particular value of the magnetic Reynolds number Re_M. Right panel: Outcome of the numerical simulations reported in this paper in the (Re, Pm) plane. The flag "YES" means that turbulence is sustained while "NO" means that turbulence decays. All runs were performed with ZEUS using a resolution $(128, 200, 128)$, exept for cases appearing within a sqared box for which the resolution was doubled.

4   Conclusion

In local numerical simulations of MRI–induced MHD turbulence, α is an increasing function of the magnetic Prandtl number Pm. This is true both in the absence of mean field, as shown in the present paper, AND in the presence of such a mean field (Lesur & Longaretti 2007). In the former case, we also find that MHD turbulence is not sustained at low Pm.

However, it is important to stress that these results were obtained at fairly low Reynolds numbers. Their extrapolation to real accretion disks is uncertain. Thus, at the present time, the value of α in accretion disks remains unknown.

References

Balbus, S. & Hawley,J., 1991, ApJ, 376, 214

Brandenburg, A. & Dobler,J., 2002, Comput. Phys. Commun., 147, 471

Fromang, S. & Papaloizou,J., 2007, A&A, 476, 1113

Fromang, S. et al., 2007, A&A, 476, 1123

Gardiner,T. & Stone,J., 2005, AIPC, 784, 475

Hawley, J. & Stone,J., 1995, Comput. Phys. Commun., 89, 127

Lesur,G. & Longaretti,P.–Y., 2007, MNRAS, 378, 1471

Papaloizou, J. et al., 2004, MNRAS, 350, 829

Ziegler, U. & Yorke,H., 1997, Comput. Phys. Commun., 101, 54