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Numerical simulations of MHD turbulence in accretion disks

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Abstract. The magnetorotational instability (MRI) is the most likely source of MHD turbulence in accretion disks. Recently, it has been realized that microscopic diffusion coefficients (viscosity and resistivity) are important in determining the saturated state of the turbulence and thereby the rate of angular momentum transport within the disc. In this paper, we present a set of numerical simulations of MRI–induced MHD turbulence in which we investigate the dependence of α, the rate of angular momentum transport, on these coefficients. We present various methods used to control the relative amount of physical versus numerical dissipation in such calculations. The results of the simulations show that α is an increasing function of the magnetic Prandtl number Pm, the ratio of viscosity over resistivity. In the absence of a mean field, we also find that MRI–induced MHD turbulence decays at low Pm.

1. Introduction

In accretion disks, turbulence is believed to be the main driver of outward angular momentum transport. For decades, its origin has been the central issue in accretion disk theory. Numerical simulations, analytic arguments and experiments all suggest that purely hydrodynamic processes are unlikely to be responsible for it and it is now widely accepted that MRI–driven MHD turbulence (Balbus & Hawley 1991) is the best candidate to play that role. Recent numerical results (Fromang & Papaloizou 2007, Fromang et al. 2007, Lesur & Longaretti 2007) have shown that microscopic dissipation coefficients (viscosity and resistivity) are important in determining the saturation properties of the turbulence in local simulations. Both have been overlooked until these last few years, largely because of the large computational resources they require. Of course, for such simulations to be physically meaningful, explicit dissipation should dominate over the numerical dissipation produced by the numerical scheme. This is another difficulty when performing these simulations. In the first part of this paper, we focus on this issue before summarizing the results obtained so far in the zero net flux case relevant to dynamo theory.

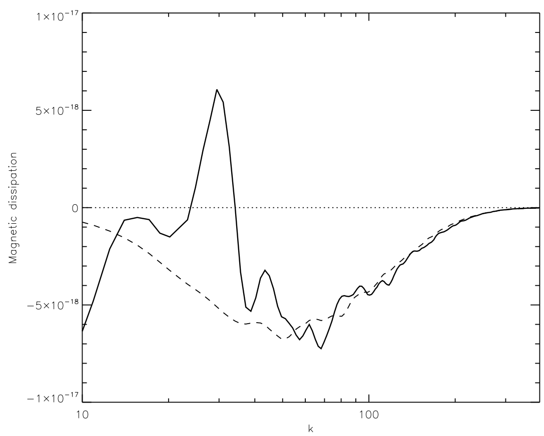

Figure 1. The solid line shows the amount of numerical dissipation produced by ZEUS in a simulation in which explicit dissipation coefficients are not included. It is plotted in spectral space. It is compared with the dissipation that would result from an explicit resistivity η. The good match at large k between the solid and the dashed line provides a way to estimate the numerical magnetic Reynolds number Re_M of the code. When the resolution is 128 cells per scale height, we obtained $Re_M \sim 30000$.

2. Monitoring numerical dissipation

In this section, we present three different methods we used to assess the amount of explicit dissipation that is needed to overcome numerical dissipation.

2.1. Numerical resistivity

A first method to quantify the relative importance of numerical and explicit dissipation is to estimate the amount of numerical dissipation itself. Fromang & Papaloizou (2007) presented a method with which numerical resistivity can be measured. It relies on monitoring in spectral space the various terms appearing in the equation describing the time evolution of the magnetic energy (such an equation is simply derived by considering the dot product of the induction equation with the magnetic field vector). In quasi steady state, the residual of all the terms, which should ideally vanish, provides a measure of numerical dissipation as a function of the wavenumber k. Such a variation is shown on figure 1 with the solid line for the case of a simulation, performed using the code ZEUS (Hawley & Stone 1995), that uses 128 cells per disk scale height H (and performed in the absence of explicit dissipation). Note that the data have been averaged over more than 200 dynamical times. The dashed line on the same figure shows the dissipation that would results from an explicit resistivity which value η is such that $Re_M = c_s H/\eta \sim 30000$, where c_s is the sound speed. The good match at large wavenumbers suggest that Re_M^{num} can be identified as a numerical magnetic Reynolds number. It is likely that the numerical Reynolds number is of the same order of magnitude. Any simulation including explicit dissipation at that resolution should therefore use viscosity and resistivity cor-

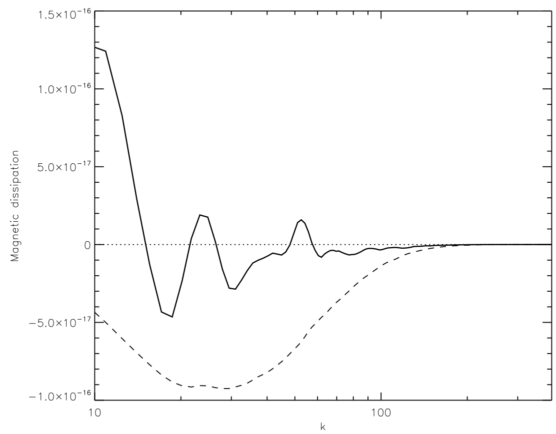

Figure 2. Same as figure 1 but for a case in which explicit dissipation coefficients are included, such that $Re_M = 3125$ and $Pm = 4$. The solid line measure numerical dissipation while the dashed lines represents the amount of explicit dissipation as a function of k. At large wavenumbers $k \geq 30$, explicit dissipation dominates indicating that numerical dissipation should play little role in determining the outcome of the simulations (the oscillations seen in numerical dissipation at small k are likely statistical errors, see text).

responding to Reynolds (magnetic or not) numbers significantly smaller than 30000. Given the results reported here, this would provide some confidence that explicit dissipation dominates over numerical dissipation. There is however a word of caution that should be added: numerical dissipation depends on the flow itself. Including explicit dissipation will change that flow and might change the value for the numerical magnetic Reynolds number. This is why the present method should only be used as a first guide and should be completed by either one (if not both) of the methods presented below.

2.2. Physical vs. numerical resistivity

A second method that is not prone to the problem described just above is to perform the same evaluation using the proper direct numerical simulation, as presented by Fromang et al. (2007). In that case, explicit dissipation can be properly monitored and compared with the remaining numerical dissipation (evaluated again as the residual of all other terms). This is done in figure 2 for a case in which $Re_M = 12500$ and $Pm = \nu/\eta = 4$, where ν is the viscosity. Numerical dissipation is shown using the solid line while the dashed line represents explicit dissipation. When $k \geq 30$, the latter is seen to dominate in absolute values. This shows that numerical dissipation should not affect the results of the simulation in that case, as it is largely smaller than explicit dissipation. At small wavenumbers, there are however large oscillations in the numerical dissipation with amplitudes comparable to the amount of explicit dissipation at these wavenumbers. Such oscillations cast some doubts onto the effect of numerical dissipation at large scales. However, recent results have suggested that they are most probably largely due to statistical errors (Simon & Hawley 2008).

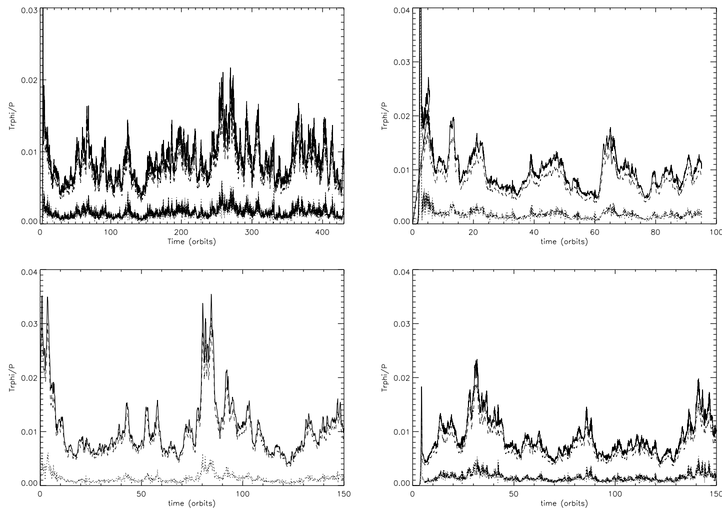

Figure 3. Time history of α in four identical local simulations of MRI–induced MHD turbulence (all using $Re = 3125$, $Pm = 4$) performed using four different codes: ZEUS, NIRVANA, a spectral code and the PENCIL code (from top left to bottom right). Each panel shows α (*solid line*), its magnetic (*dashed line*) and its hydrodynamic parts (*dotted line*). The good agreement obtained when comparing the four simulations indicates that explicit dissipation dominates over numerical dissipation.

2.3. Code comparison

The last method we present is a code comparison, in which we reproduce exactly the same simulation using four different codes having different amount of numerical dissipation. These codes are ZEUS, NIRVANA (Ziegler & Yorke 1997), the PENCIL code (Brandenburg & Dobler 2002) and a spectral code (Lesur & Longaretti 2007). In all cases, we used $Re_M = 12500$ and $Pm = 4$, as in section 2.2. above. The results of the four simulations are in very good agreement, as shown on figure 3 where we plot for all cases the rate of angular momentum, the so called α parameter (see Fromang et al. 2007 for a definition of α), as a function of time. The scatter in the time averaged α value is only about 10% (Fromang et al. 2007). This is another good indication that numerical dissipation does not affect the saturation level of MHD turbulence in that case.

3. Parameter space study

Having shown how to quantify numerical dissipation, we can now safely vary the dissipation coefficients (and accordingly, the resolution when needed) and explore the variation of the turbulent properties as ν and η are changed. This is the purpose of this section. We use ZEUS and study the variation of α with the Reynolds and magnetic Prandtl numbers.

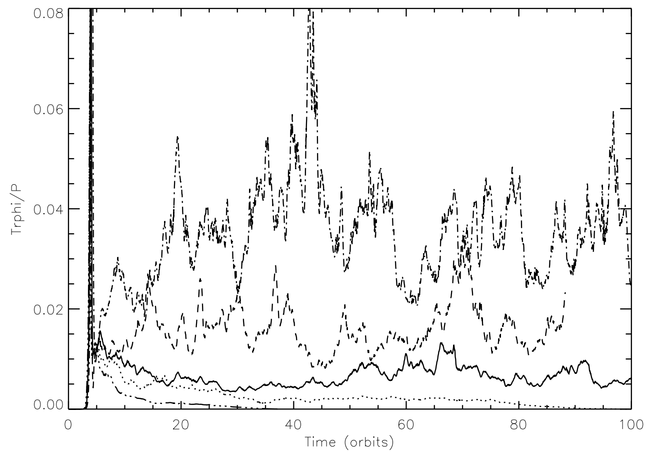

Figure 4. Time history of α in the case $Re_M = 12500$ and $Pm = 16$ (*dotted–dashed line*), $Pm = 8$ (*dashed line*), $Pm = 4$ (*solid line*), $Pm = 2$ (*dotted line*) and $Pm = 1$ (*dotted–dotted–dashed line*). The angular momentum transport increases with the magnetic Prandtl number Pm and vanishes when $Pm \leq 2$ for this particular value of the magnetic Reynolds number Re_M.

3.1. The effect of the Prandtl number

The first outcome of our simulations is the strong dependence of α on the magnetic Prandtl number Pm. This can be demonstrated by using a set of simulations with different values of Pm. Figure 3 shows the time history of α for such a sequence. We report the results of six simulations having $Pm = 16$ (*dotted–dashed line*), $Pm = 8$ (*dashed line*), $Pm = 4$ (*solid line*), $Pm = 2$ (*dotted line*) and $Pm = 1$ (*dotted–dotted–dashed line*). In all simulations, the resistivity is such that $Re_M = c_s H/\eta = 12500$. Figure 2 shows that α is an increasing function of Pm. This trend has also been reported in net flux simulations of the MRI (Lesur & Longaretti 2007). In the absence of a mean magnetic field, our results also demonstrate that MHD turbulence decays for values of Pm smaller than about two when $Re_M = 12500$.

3.2. The overall results

Finally, we report in figure 5 the results of all the simulations we performed. Both dimensionless numbers $Re = c_s H/\nu$ and Re_M are varied. Figure 5 describes the nature of the flow in the (Re, Re_M) plane: "YES" means that MHD turbulence is sustained while "NO" corresponds to cases in which it is found to decay. For each Re, there is a critical value of Pm, Pm_c, below which turbulence is not sustained (for example, in section 4, we found that $Pm_c \sim 2$ when $Re = 3125$). For the range of Re probed in these simulations, Pm_c is a decreasing function of Re. We found that turbulence was never sustained when $Pm \leq 1$. However, it should be stressed that we could not isolate any asymtoptic regime while performing these simulations. It is thus dangerous to extrapolate these results to values of the dissipation coefficients different than those reported in this paper.

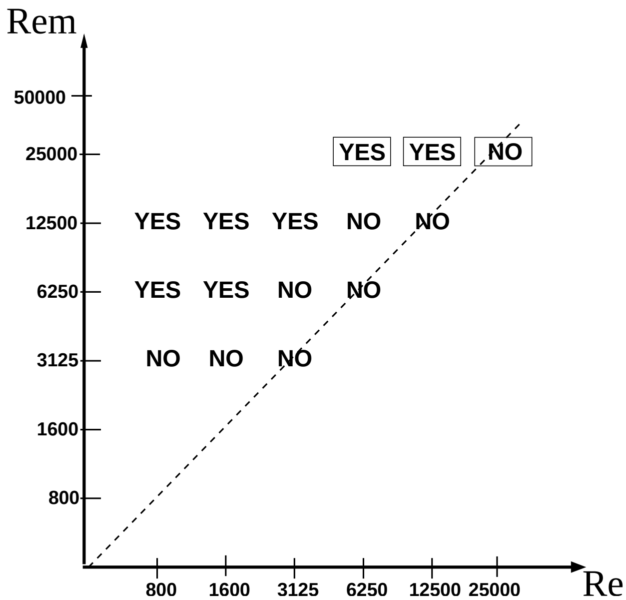

Figure 5. Outcome of the numerical simulations reported in this paper in the (Re, Pm) plane. The flag "YES" means that turbulence is sustained while "NO" means that turbulence decays. All runs were performed with ZEUS using a resolution $(128, 200, 128)$, except for cases appearing within a squared box for which the resolution was doubled.

4. Conclusion

In this paper, we have detailed different methods that can be used to monitor the relative importance of numerical and explicit dissipation when performing local numerical simulations of MRI–induced MHD turbulence. These methods makes it possible to perform a systematic study of the saturated state of the turbulence as dimensionless numbers are varied. We found a strong dependence of α with the magnetic Prandtl number, but have failed to reach any asymptotic limit when decreasing viscosity and resistivity. At the present stage, it is thus impossible to extrapolate our results to real accretion discs in which the value of α remains largely unknown.

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